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Energy efficiency optimization of one-way and two-way DF relaying considering circuit power

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Abstract In this paper, the energy efficiency (EE) of a decode and forward (DF) relay system is studied, where two sources communicate through a half-duplex relay node in one-way and two-way relaying strategies. Both the circuitry power and the transmission power of all nodes are taken into consideration. In addition, three different coding schemes for two-way DF relaying strategy with two phases and two-way DF relaying with three phases are considered. The aim is to maximize the EE of the system for a constant spectral efficiency (SE). For this purpose, the transmission time and the transmission power of each node are optimized. Simulations are used to compare the EE-SE curve of different DF strategies with one-way and two-way amplify and forward (AF) strategies and direct transmission (DT), to find the best energy efficient strategy in different SE conditions. Analytical and simulation results demonstrate that in low SE conditions, DF relaying strategies are more energy efficient compared to that of AF strategies and DT. However, in high SE conditions, the EE of two-way AF relaying and DT strategy outperform some of the DF relaying strategies. In simulations, the impact of different circuitry power and different channel conditions on the EE-SE curves are also investigated.

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¹ Department of Electrical and Computer Engineering, Isfahan University of Technology, Isfahan 84156-83111, Iran **Keywords** Energy efficiency · Optimization · Decodeand-forward · Two-way relaying · Circuit power

1 Introduction

In recent years, attempts are made to find energy efficient methods for different layers of a communication network. As a result, a great thorough framework, green communication, has emerged to save energy in modern systems and standards. Four major trade-offs have been proposed for green radio in [1]: Energy efficiency-Deployment efficiency, Energy efficiency-Spectral efficiency, Bandwidth-Power and Delay-Power.

In traditional literature, the EE was defined as "information bits per unit of transmit energy", however, practical concerns result in taking circuit energy consumption into account for the energy consumption model (ECM). The EE metric has been redefined as "information bits per unit of consumed energy (not just transmit energy)", where an additional circuit power factor needs to be considered [2]. Also, spectral efficiency (SE) defined as system throughput per unit of bandwidth, is a widely accepted performance indicator of wireless networks. There is a trade-off between maximizing SE and EE in communication systems, therefore, it is important to balance the two metrics in future communication networks [1]. In [2], it is shown that consideration of the circuit power can affect the traditional EE-SE trade off in point to point transmission systems. It turns the EE-SE curve from a cup shape to a bell shape. In other words, with the consideration of circuit power, a point can be found in the EE-SE curve for which SE is none zero and EE is maximized [1, 2].

In [3], authors analysed the best modulation strategy to minimize the total energy needed to transmit a given

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number of bits. Their power consumption model (PCM) involves transmit power besides constant circuit power of nodes. They provide a clear and through explanation to justify the modelling of the transmitter and receiver circuit power as a constant factor in PCM. It is reasonable to consider only the transmission energy in the long-range applications because in these cases, the transmission energy dominates the circuit energy in the ECM. However, in short-range applications such as sensor networks, the circuit energy of the devices is comparable or even dominates the transmission energy [4]. In [4] authors analysed the best modulation and transmission strategy to find the most energy efficient scheme to send a given number of bits. Moreover, in [3-9], the PCMs include circuit power of nodes. It is shown that as a practical point of view the circuit power should be considered since it can change various aspects of performance measurements in a communication network (such as EE and best modulation scheme).

The limited power and bandwidth resources in communication networks, and the multipath fading nature of wireless systems motivate designers to use the idea of cooperative relaying [10, 11]. Relaying and cooperative communication are also mentioned as a promising architecture to improve EE. In [12], authors address the energy efficiency of cellular network communication systems, green metrics, and emerging technologies such as cognitive radio and cooperative relaying for obtaining green networks. It is shown that by the increase of radio access points (RAPs) in a network (relays are good examples of RAPs in wireless networks) the energy consumption of the network (without consideration of the circuitry power) will be improved [10]. In [13], the benefits of relay based systems for improving EE is studied. It is shown that when only casual channel state information (CSI) is available or if the channel is not rich in diversity, such as Rayleigh and Rician channels, only relay cooperative schemes with adaptive power allocation will meet the expected energy bounded. In this case, authors do not consider the circuitry power of devices in the PCM. In [14], authors study the EE of direct transmission in comparison with one-way relaying. They maximize both packet size and modulation level jointly. Also the EE of cooperative beam forming based transmission is studied in [15]. A set of nodes using DF scheme is assumed as relays to cooperate with source node. In the PCM, the overhead energy consumption for obtaining the CSI is also modelled. In [1, 2, 12, 16] the relay is conceived as a candidate for improving the EE in communication networks when the circuit power is also considered in the PCM. However, they did not provide a tangible proof or graph to conclude that how relay cooperative communication can be helpful to make the EE get better.

Two-way relaying is proposed as a cooperative scheme to improve the SE of one-way relaying. One-way relaying leads to the loss of SE due to the pre-log factor one-half in the corresponding capacity expression, however, two-way relaying avoids the pre-log factor one-half still uses half duplex equipment [17]. For two-way relaying strategy, several schemes have been proposed including AF, DF and a new approach based on lattice codes called functional decode and forward (FDF) [18, 19]. In [19], different twoway relaying strategies are introduced and studied to find the achievable bit rates. Also, the capacity gap of two-way AF, DF and FDF relaying are compared with the upper bound capacity of two-way relaying channels. In [20, 21], MIMO cooperative two-way relaying is analyzed and they both consider the DF case. In [20], the transmit covariance matrix optimization of MIMO Gaussian bidirectional broadcast channel is studied and in [21], the impact of transmit CSI in two different re-encoding schemes including superposition coding and XOR precoding is investigated. In [22, 23], authors maximize the sum-rate of a network with minimum power consumption in a DF twoway relaying network. In [24], authors propose a practical power allocation technique in a two-way AF (TWAF) relaying network for a wide range of signal-to-noise ratio (SNR) conditions. However, the circuitry power is not considered in the PCM of the mentioned two-way relaying systems.

In [25], authors consider a system with two sources and one half-duplex relay node working with AF scheme. They consider both transmission and circuitry energy in the ECM and optimize the transmission time and the transmission powers at each node to achieve the best EE for a given number of bits. Then, the optimum EE of three strategies including one-way AF relaying (OWAF), TWAF and direct transmission (DT) are compared. In [26], the model is extended to OFDM AF relay systems and the active number of subcarriers and the number of bits assigned to each subcarrier at two source nodes are optimized. In [27], we solved the same optimization problem for one-way DF relaying (OWDF).

In this paper, we consider five different relaying cases including OWDF, two-way upper bound relaying (TWUB), two-way FDF relaying (TWFDF), two-way DF relaying (TWDF), and two-way three phases DF relaying (TW3DF). As it was mentioned before, both the transmission and circuitry energy of the nodes are considered in the ECM. This paper has three contributions:

- 1. We optimize the transmission time and the transmission power of five DF relaying strategies (OWDF, TWUB, TWFDF, TWDF, and TW3DF) to achieve the optimal circuit power-considered energy efficiency (CPEE).
- We compare the CPEE of eight different possible strategies including OWDF, OWAF, TWUB, TWFDF, TWDF,TW3DF, TWAF and DT in different SE

conditions. It is shown that under different SE conditions which strategy provides the best CPEE.

3. Also, the impact of channel condition and different circuit power on the CPEE of DF strategies are illustrated in different SE conditions.

2 System model

We consider a relay network consisting of two sources and one fixed relay working in DF mode. The system is delay constrained which has a hard deadline duration time of *T*. In each block, nodes s_1 and s_2 transmit B_1 and B_2 bits, respectively to each other through a relay node, *R*. The first direction is $s_1 \rightarrow s_2$ and the second direction is $s_2 \rightarrow s_1$.

The wireless channel considered in each direction is a W-Hertz frequency-flat channel. The noise model is an additive white Gaussian noise (AWGN) with the power spectral density of N_0 . The perfect CSI of the all links are available at each node. The channel coefficients for $s_1 \rightarrow R$, $s_2 \rightarrow R$ and $s_1 \rightarrow s_2$ are shown by h_1, h_2, h_{sls2} , respectively. We assume that these channels are slow varying, therefore, they are constant in a block duration of *T*.

A prominent practical aspect of this paper is the consideration of the circuit power of each node. Based on this assumption, each node has three modes: transmission, reception and idle mode. Each node may work as a transmitter or receiver and it is not necessary to operate in all duration time of *T*. The maximum available transmission power in each node is P_{max}^t .

According to [3], we just consider the power of radio frequency (RF) chain as a circuit power which is independent of the bit rate. The circuitry power of transmission, reception and idle mode are shown by P^{ct} , P^{cr} and P^{ci} , respectively. According to [25], $P^{ct} = P^{cr}$ and it is assumed that the circuit power in transmission or reception mode is much larger than the idle mode $P^{ct} = P^{cr} > P^{ci}$. Also, all the circuit powers in different modes are assumed to be constant and bit rate independent [25].

3 One-way DF relaying

In this section, energy efficiency of one-way DF relaying (OWDF) scheme in a block duration of T is developed. As it is shown, the energy of the system is a function of the transmission time and power, which are related to each other by the capacity expression of the system.

In the following, it is shown that because of the constant transmission bits, maximizing the energy efficiency, is equivalent to minimizing the system energy consumption. To this aim, first the summation of transmission power is minimized and derived as a function of transmission time and then the energy of the system will be optimized with respect to the transmission time only.

3.1 CPEE of OWDF in a block duration of T

In OWDF scenario as depicted in Fig. 1(b), the transmission process contains four phases. In the first phase s_1 sends B_1 bits to the relay node. Then, node R decodes the received bits, encodes the new message and sends it to s_2 in the second phase. Then, s_2 sends B_2 bits to the relay node and relay sends these bits to s_1 , respectively in the third and fourth phases. It is assumed that $\frac{T_{O1}}{2}$ seconds is spend in each of the first two phases and also $\frac{T_{O2}}{2}$ seconds in each of the last two phases. As it is clear $T_{O1} + T_{O2} < T$, so the system is in idle mode for $T - T_{O1} - T_{O2}$ s. Therefore, the CPEE for one way relaying is derived as:

$$\eta_{EEO} = \frac{B_1 + B_2}{E_O},\tag{1}$$

$$\begin{split} E_{O} &= \frac{T_{O1}}{2} \left(\frac{P_{s1}^{T}}{\epsilon} + P_{s1}^{ct} + P_{r}^{cr} + P_{s2}^{ci} + \frac{P_{r1}^{T}}{\epsilon} + P_{r}^{ct} + P_{s2}^{cr} + P_{s1}^{ci} \right) \\ &+ \frac{T_{O2}}{2} \left(\frac{P_{s2}^{T}}{\epsilon} + P_{s2}^{ct} + P_{r}^{cr} + P_{s1}^{ci} + \frac{P_{r2}^{T}}{\epsilon} + P_{r}^{ct} + P_{s1}^{cr} + P_{s2}^{ci} \right) \\ &+ (T - T_{O1} - T_{O2})(P_{s1}^{ci} + P_{s2}^{ci} + P_{r}^{ci}), \\ E_{O} &= T_{O1} \left(\frac{P_{s1}^{T} + P_{r1}^{T}}{2\epsilon} + P_{O}^{c1} - P_{O}^{ci} \right) \end{split}$$

$$+T_{O2}\left(\frac{P_{s2}^{T}+P_{r2}^{T}}{2\varepsilon}+P_{O}^{c2}-P_{O}^{ci}\right)+TP_{O}^{ci},$$
(2)

where
$$P_{O}^{c1} \triangleq \frac{(P_{s1}^{c1} + P_{r}^{c1} + P_{s2}^{c1} + P_{s2}^{c1} + P_{s2}^{c1} + P_{s2}^{c1} + P_{s1}^{c2} + P_{s1}^{c1})}{2}$$
, $P_{O}^{c2} \triangleq \frac{(P_{s2}^{c1} + P_{r}^{c1} + P_{s1}^{c1} + P_{s2}^{c1} + P_{s1}^{c1})}{2}$, $P_{O}^{c2} \triangleq P_{s1}^{c1} + P_{s2}^{c1} + P_{r}^{c1}$, also P_{r1}^{T} and



(d) Two-way relaying with three phases

Fig. 1 Transmission procedure in a block duration of T. **a** Direct transmission, **b** one-way relaying, **c** two-way relaying with two phases, **d** two-way relaying with three phases

 P_{r2}^T are the relay transmission power in the first and second direction, respectively and $\varepsilon \in (0, 1]$ indicates the power amplifier efficiency. It is obvious that in (1) maximizing energy efficiency is equivalent to minimizing the energy of the system when $B_1 + B_2$ is constant. The circuit power is constant for each node, hereby, the optimization problem can be defined as:

With this assumption the capacity expression will be obtained as:

$$C_{1OWDF} = \frac{B_1}{T_{O1}} = \frac{W}{2} \log_2 \left(1 + \frac{|h_2|^2 P_{r_1}^T}{N_0} \right).$$
(8)

So the transmission power of relay and node s_1 are derived as:

$$\begin{array}{ll}
\min_{\substack{T_{O1}, T_{O2}, P_{s1}^{T}, P_{s2}^{T}, P_{r}^{T}} & T_{O1} \left(\frac{P_{s1}^{T} + P_{r1}^{T}}{2\varepsilon} + P_{O}^{c1} - P_{O}^{ci} \right) + T_{O2} \left(\frac{P_{s2}^{T} + P_{r2}^{T}}{2\varepsilon} + P_{O}^{c2} - P_{O}^{ci} \right) + TP_{O}^{ci} \\
s.t & T_{O1} + T_{O2} \le T, \quad P_{s1}^{T} \le P_{\max}^{t}, \quad P_{s2}^{T} \le P_{\max}^{t}, \quad P_{r}^{T} \le P_{\max}^{t}.
\end{array} \tag{3}$$

3.2 Minimizing the summation of transmission power

In this subsection, we minimize the transmit power subject to constant transmission time and derive the solution as a function of transmission time using the capacity expression of the system.

The capacity of each direction employing DF scheme for relaying is obtained as [28]:

$$C_{1OWDF} = \frac{B_1}{T_{O1}} = \frac{W}{2} \min\left\{ \log_2\left(1 + \frac{|h_1|^2 P_{s1}^T}{N_0}\right), \quad (4) \\ \log_2\left(1 + \frac{|h_2|^2 P_{r1}^T}{N_0}\right)\right\}, \quad (4) \\ C_{2OWDF} = \frac{B_2}{T_{O2}} = \frac{W}{2} \min\left\{ \log_2\left(1 + \frac{|h_2|^2 P_{s2}^T}{N_0}\right), \\ \log_2\left(1 + \frac{|h_1|^2 P_{r2}^T}{N_0}\right)\right\}. \quad (5)$$

Our objective function which minimizes the total transmission power in each direction can be formulated as follows:

$$\begin{array}{ll} \min \\ P_{s1}^{T}, P_{r1}^{T} & P_{s1}^{T} + P_{r1}^{T} \\ s.t. & P_{s1}^{T} \le P_{\max}^{t}, \quad P_{r1}^{T} \le P_{\max}^{t}, \quad (4). \end{array}$$

$$\begin{array}{ll}
\min_{\substack{P_{s2}^T, P_{r2}^T \\ s.t}} & P_{s2}^T + P_{r2}^T \\ P_{s2}^T \le P_{\max}^t, & P_{r2}^T \le P_{\max}^t, \quad (5). \end{array}$$

For brevity, we just solve problem (6). Problem (7) can also be solved with the same solution.

We consider two situations.

1. if
$$|h_1|^2 P_{s1}^T \ge |h_2|^2 P_{r1}^T$$
:

$$P_{r1}^{T} = \frac{\left(2^{\frac{2B_{1}}{T_{O1}W}} - 1\right)N_{0}}{\left|h_{2}\right|^{2}},$$
(9a)

$$P_{s1}^{T} \ge P_{r1}^{T} \left(\frac{|h_{2}|^{2}}{|h_{1}|^{2}}\right) = \frac{\left(2^{\frac{2B_{1}}{T_{01}W}} - 1\right)N_{0}}{|h_{1}|^{2}}.$$
(9b)

Therefore, problem (6) is solved as:

$$P_{s1opt}^{T} + P_{r1opt}^{T} = \left(2^{\frac{2B_{1}}{T_{O1}W}} - 1\right) N_{0} \left(\frac{1}{|h_{1}|^{2}} + \frac{1}{|h_{2}|^{2}}\right)$$
$$= \frac{\left(2^{\frac{2B_{1}}{T_{O1}W}} - 1\right) N_{0}}{|h_{effOWDF}|^{2}}, \qquad (10)$$
where $\frac{1}{(1-1)^{2}} \triangleq \left(\frac{1}{(1-2)^{2}} + \frac{1}{(1-2)^{2}}\right).$

where
$$\frac{1}{|h_{effOWDF}|^2} \stackrel{\simeq}{=} \left(\frac{1}{|h_1|^2} + \frac{1}{|h_2|^2}\right)$$

2. *if* $|h_1|^2 P_{s1}^T \le |h_2|^2 P_{r1}^T$:

With this assumption, the capacity expression is obtained as:

$$C_{1OWDF} = \frac{B_1}{T_{O1}} = \frac{W}{2} \log_2 \left(1 + \frac{|h_1|^2 P_{s_1}^T}{N_0} \right).$$
(11)

The powers of the nodes are derived as:

$$P_{s1}^{T} = \frac{\left(2^{\frac{2B_{1}}{T_{01}W}} - 1\right)N_{0}}{\left|h_{1}\right|^{2}},$$
(12a)

$$P_{r1}^{T} \ge P_{s1}^{T} \left(\frac{|h_{1}|^{2}}{|h_{2}|^{2}}\right) = \frac{\left(2^{\frac{2B_{1}}{T_{O1}W}} - 1\right)N_{0}}{|h_{2}|^{2}}.$$
 (12b)

It is obvious that both situations lead to the same solution (Eq. (10)) for the given optimization problem in (6).

Similar to (6), the optimum transmission power of the nodes in the second direction, the answer of problem (7), can be obtained as:

$$P_{s2opt}^{T} + P_{r2opt}^{T} = \left(2^{\frac{2B_{2}}{T_{O2}W}} - 1\right) N_{0} \left(\frac{1}{|h_{2}|^{2}} + \frac{1}{|h_{1}|^{2}}\right)$$
$$= \frac{\left(2^{\frac{2B_{2}}{T_{O2}W}} - 1\right) N_{0}}{|h_{effOWDF}|^{2}}.$$
(13)

To satisfy all the constraints of problems (6) and (7), it should be guaranteed that C_{10WDF} and C_{20WDF} are less than the maximum data rates supported by the maximum transmit power. So the constraints of (6) and (7) lead to set a minimum transmission time for the system which can be derived as follows:

$$T_{O1\min} = \frac{B_1}{\frac{W}{2}\min\left\{\log_2\left(1 + \frac{|h_1|^2 P_{\max}'}{N_0}\right), \ \log_2\left(1 + \frac{|h_2|^2 P_{\max}'}{N_0}\right)\right\}},$$
(14)

$$T_{O2\min} = \frac{B_2}{\frac{W}{2}\min\left\{\log_2\left(1 + \frac{|h_2|^2 P'_{\max}}{N_0}\right), \log_2\left(1 + \frac{|h_1|^2 P'_{\max}}{N_0}\right)\right\}}.$$
(15)

Minimum transmission times are used as constraints in the optimization problem.

3.3 Optimization of CPEE in OWDF relaying

Here the minimum power transmissions (10) and (13) are substituted in the energy optimization problem given in (3) and the optimization problem is derived based on transmission times only.

Since the objective function is convex, the optimum transmission time can be derived when first order derivative is set to zero.

$$\frac{d}{dT_{O1}} \left(T_{O1} \left(\frac{\left(2^{\frac{2B_1}{T_{O1}W}} - 1\right) N_0}{2\varepsilon \left| h_{effOWDF} \right|^2} + P_O^{c1} - P_O^{ci} \right) \right) = 0,$$

$$\left[\frac{\left(2^{\frac{2B_1}{T_{O1}W}} - 1\right) N_0}{2\varepsilon \left| h_{effOWDF} \right|^2} + P_O^{c1} - P_O^{ci} \right] - \frac{2^{\frac{2B_1}{T_{O1}W}} N_0}{2\varepsilon \left| h_{effOWDF} \right|^2} \frac{2B_1}{T_{O1}W} \ln 2$$

$$= 0|_{T_{O1} = T_{O1opt}}.$$
(18)

where T_{Olopt} is the optimum transmission time in the first direction. It is difficult to find a closed-form solution for the optimum transmission time. However, Eq. (18) is solvable and we can conclude that:

$$\begin{bmatrix} \left(2^{\frac{2B_{1}}{T_{Olopt}W}} - 1\right)N_{0} \\ \hline 2\varepsilon |h_{effOWDF}|^{2} + P_{O}^{c1} - P_{O}^{ci} \end{bmatrix} \\
= \frac{2^{\frac{2B_{1}}{T_{Olopt}W}}N_{0}}{2\varepsilon |h_{effOWDF}|^{2}} \frac{2B_{1}}{T_{Olopt}W} \ln 2.$$
(19)

After finding the optimum transmission time in both directions ($T_{O1opt} = T_{O2opt} = T_{opt}$), the minimum energy is obtained and so the maximum energy efficiency is derived:

$$\begin{array}{ll}
\min_{T_{O1}, T_{O2}} & T_{O1} \left(\frac{\left(2^{\frac{2B_{1}}{T_{O1}W}} - 1\right) N_{0}}{2\varepsilon \left|h_{effOWDF}\right|^{2}} + P_{O}^{c1} - P_{O}^{ci}\right) + T_{O2} \left(\frac{\left(2^{\frac{2B_{2}}{T_{O2}W}} - 1\right) N_{0}}{2\varepsilon \left|h_{effOWDF}\right|^{2}} + P_{O}^{c2} - P_{O}^{ci}\right) + TP_{O}^{ci} \\
s.t & T_{O1} + T_{O2} \leq T, \ T_{O1} \geq T_{O1\min}, \ T_{O2} \geq T_{O2\min}.
\end{array}$$
(16)

According to [29], the optimization problem in (16) is a convex optimization problem (the second order derivative with respect to T_{O1} and T_{O2} is positive). So this convex optimization problem can be solved with methods of convex optimization in [29]. However, if we consider $B_1 = B_2 = B$, transmission times in both directions will be obtained identically ($T_{O1} = T_{O2}$). Because of the symmetric conditions and balanced information transmission, it is sufficient to find the optimized transmission time in one direction. In the first direction the problem is defined as:

$$\min_{T_{O1}} T_{O1} \left(\frac{\left(2^{\frac{2B_1}{T_{O1}W}} - 1 \right) N_0}{2\varepsilon \left| h_{effOWDF} \right|^2} + P_O^{c1} - P_O^{ci} \right)$$
(17)

$$\eta_{EElow}^{OWDF} = \frac{B_1 + B_2}{E_{Oopt}}$$

$$= \frac{2B}{\frac{2BN_0(\ln 2)}{\epsilon |h_{effOWDF}|^2 W} 2^{\frac{2B}{WT_{Oopt}}} + TP_O^{ci}},$$
(20)

where E_{Oopt} is the minimum energy consumed in a block duration of *T*. It should be considered that (20) is valid for low traffic conditions. In high traffic conditions the optimum transmission time derived in (18) is larger than the time slot *T*. So all of the time duration is used for transmission and the optimum energy efficiency is derived as:

$$\eta_{EEhigh}^{OWDF} = \frac{2B}{T\left(\frac{(2^{\frac{4R}{VW}} - 1)N_0}{2\varepsilon \left|h_{effOWDF}\right|^2}\right) + \frac{T}{2}(P_O^{c1} + P_O^{c2})}.$$
(21)

4 Two-way DF relaying with two phases

In this section, the CPEE optimization is developed in twoway relaying scenario where the transmission of information in both directions occurs in two phases. According to Fig. 1(c), in the first phase, source nodes, s_1 and s_2 , send their information bits, B_1 and B_2 , to the relay node R and in the second phase, node R broadcasts its message to both sources. Three cases for two-way relaying are considered. In the first case, we assume that the upper bound capacity is achievable in two-way relaying channel, called as two-way upper bound relaying (TWUB) [19]. The second case is called two-way functional decode and forward relaying (TWFDF) [19], while the third one is two-way decode and forward relaying (TWDF).

The CPEE of the two-way relaying system in a block duration of *T* is identical for all two-way strategies with two phases. In the first phase of the transmission, source nodes send their data bits to the relay, so s_1 and s_2 are in the transmission mode and node *R* is in the reception mode. Then in the second phase, the relay sends information to the sources. It is assumed that the system spends $\frac{T_T}{2}$ seconds in each phase. Clearly, $T_T \leq T$, so the system is in idle mode for $T - T_T$ seconds and the CPEE of the system in the two-way scenario is derived as:

$$\eta_{EET} = \frac{B_1 + B_2}{E_T},\tag{22}$$

$$E_{T} = \frac{T_{T}}{2} \left(\frac{P_{s1}^{T} + P_{s2}^{T}}{\epsilon} + P_{s1}^{ct} + P_{s2}^{ct} + P_{r}^{cr} \right) + \frac{T_{T}}{2} \left(\frac{P_{r}^{T}}{\epsilon} + P_{r}^{ct} + P_{s1}^{cr} + P_{s2}^{cr} \right) + (T - T_{T})(P_{s1}^{ci} + P_{s2}^{ci} + P_{r}^{ci}), E_{T} = T_{T} \left(\frac{P_{s1}^{T} + P_{s2}^{T} + P_{r}^{T}}{2\epsilon} + P_{T}^{c} - P_{T}^{ci} \right) + TP_{T}^{ci},$$
(23)

where $P_T^c \triangleq \frac{(P_{s1}^{ct} + P_{s2}^{ct} + P_{s1}^{ct} + P_{s2}^{cr} + P_{s2}^{cr} + P_{s2}^{cr})}{2}$, $P_T^{ci} \triangleq P_{s1}^{ci} + P_{s2}^{ci} + P_r^{ci}$. Obviously maximizing the CPEE is equivalent to minimizing E_T when the transmitted data bits are constant. So the optimization problem is obtained as follow:

$$\begin{array}{ccc} \min \\ T_T, P_{s_1}^T, P_{s_2}^T, P_r^T & T_T \left(\frac{P_{s_1}^T + P_{s_2}^T + P_r^T}{2\varepsilon} + P_r^c - P_T^{ci} \right) + TP_T^{ci} \\ s.t & T_T \leq T, \ P_{s_1}^T \leq P_{\max}^t, \ P_{s_2}^T \leq P_{\max}^t, \ P_r^T \leq P_{\max}^t \\ \end{array}$$
(24)

Because of different capacity expressions the CPEE optimization is different for TWUB, TWFDF and TWDF.

4.1 CPEE optimization of TWUB

In the second step, it is desired to derive the minimum summation of the transmission power of the nodes as a function of the transmission time, while the transmission time is constant. For a given B_1 and B_2 bits and constant transmission time of T_{TWUB} the capacity expression of each direction for TWUB is derived as [19]:

$$C_{1TWUB} = \frac{B_1}{T_{TWUB}} = \frac{W}{2} \min\left\{ \log_2\left(1 + \frac{|h_1|^2 P_{s1}^T}{N_0}\right), \quad (25)\right\}$$

$$\log_2\left(1 + \frac{|h_2|^2 P_r^T}{N_0}\right) \right\}, \quad (25)$$

$$C_{2TWUB} = \frac{B_2}{T_{TWUB}} = \frac{W}{2} \min\left\{ \log_2\left(1 + \frac{|h_2|^2 P_{s2}^T}{N_0}\right), \quad (26)\right\}$$

$$\log_2\left(1 + \frac{|h_1|^2 P_r^T}{N_0}\right) \right\}.$$

In contrast to the one-way relaying system, in DF two-way relaying scenario, according to (23), the summation power of three nodes should be minimized for both directions. So the problem is derived as:

$$\begin{array}{ll}
\min_{\substack{P_{s1}^T, P_{s2}^T, P_r^T \\ s.t}} & P_{s1}^T + P_{s1}^T + P_{s2}^T + P_r^T \\
P_{s1}^T \le P_{\max}^t, P_{s2}^T \le P_{\max}^t, P_r^T \le P_{\max}^t, (25), (26).
\end{array}$$
(27)

To solve (27) we consider four possible states:

1. if
$$|h_1|^2 P_{s_1}^T \ge |h_2|^2 P_r^T$$
 and $|h_2|^2 P_{s_2}^T \ge |h_1|^2 P_r^T$:

With this assumption the power of the relay node determines the capacity expression of both directions. To obtain desired bit rates of B_1/T_{TWUB} and B_2/T_{TWUB} we have:

$$\frac{B_1}{T_{TWUB}} \leq \frac{W}{2} \log_2 \left(1 + \frac{|h_2|^2 P_r^T}{N_0} \right)
\rightarrow P_r^T \geq \frac{\left(2^{\frac{2B_1}{WT_{TWUB}}} - 1 \right) N_0}{|h_2|^2},$$
(28a)

$$\frac{B_2}{T_{TWUB}} \leq \frac{W}{2} \log_2 \left(1 + \frac{|h_1|^2 P_r^T}{N_0} \right)
\rightarrow P_r^T \geq \frac{\left(2^{\frac{2B_2}{WT_{TWUB}}} - 1 \right) N_0}{|h_1|^2},$$
(28b)

$$P_{r\min}^{T} = \max\left\{\frac{\left(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{2}|^{2}}, \frac{\left(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{1}|^{2}}\right\}.$$
(28c)

The transmission powers of other nodes are derived as:

$$P_{s1}^{T} \ge \frac{P_{r}^{T} |h_{2}|^{2}}{|h_{1}|^{2}} \to P_{s1\min}^{T} \ge \frac{P_{r\min}^{T} |h_{2}|^{2}}{|h_{1}|^{2}},$$
(29a)

$$P_{s2}^{T} \ge \frac{P_{r}^{T}|h_{1}|^{2}}{|h_{2}|^{2}} \to P_{s2\min}^{T} \ge \frac{P_{r\min}^{T}|h_{1}|^{2}}{|h_{2}|^{2}}.$$
 (29b)

So the optimum solution for (27) is:

$$P_{s1\min}^{T} + P_{s2\min}^{T} + P_{r\min}^{T} \ge \left(\frac{|h_{2}|^{2}}{|h_{1}|^{2}} + \frac{|h_{1}|^{2}}{|h_{2}|^{2}} + 1\right)$$

$$\max\left\{\frac{\left(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{2}|^{2}}, \frac{\left(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{1}|^{2}}\right\}.$$
(30)

In "Appendix" it is shown that (30) can be written as:

$$P_{\min TWUB}^{I}(T_{TWUB}) = P_{s1\min}^{I} + P_{s2\min}^{I} + P_{r\min}^{I},$$

$$P_{\min TWUB}^{T}(T_{TWUB}) = \frac{\left(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{1}|^{2}} + \frac{\left(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{2}|^{2}}$$

$$+ \max\left\{\frac{\left(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{2}|^{2}}, \frac{\left(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{1}|^{2}}\right\}.$$
(31)

2. if
$$|h_1|^2 P_{s1}^T \le |h_2|^2 P_r^T$$
 and $|h_2|^2 P_{s2}^T \ge |h_1|^2 P_r^T$:

With this assumption, the capacity expressions of the first and second direction are determined by P_{s1}^T and P_r^T , respectively. Therefore, the optimum power of each node is derived as:

$$\frac{B_1}{T_{TWUB}} \le \frac{W}{2} \log_2 \left(1 + \frac{|h_1|^2 P_{s1}^T}{N_0} \right) \to P_{s1\min}^T \ge \frac{\left(2^{\frac{2B_1}{WT_{TWUB}}} - 1 \right) N_0}{|h_1|^2},$$
(32a)

$$\frac{B_2}{T_{TWUB}} \le \frac{W}{2} \log_2 \left(1 + \frac{|h_1|^2 P_r^T}{N_0} \right)
\rightarrow P_{r\min}^T \ge \frac{\left(2^{\frac{2B_2}{WT_{TWUB}}} - 1 \right) N_0}{|h_1|^2},$$
(32b)

$$P_{s2}^{T} \ge \frac{P_{r}^{T} |h_{1}|^{2}}{|h_{2}|^{2}} \to P_{s2\min}^{T} = \frac{\left(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1\right) N_{0}}{|h_{2}|^{2}}.$$
 (32c)

Using the assumption of this state we have:

$$|h_{1}|^{2} P_{s1}^{T} \leq |h_{2}|^{2} P_{r}^{T} \rightarrow \frac{\left(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1\right) N_{0}}{|h_{2}|^{2}} \leq \frac{\left(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1\right) N_{0}}{|h_{1}|^{2}}.$$
(33)

Clearly in this state, the optimum solution for (27) is (31) again.

3. *if*
$$|h_1|^2 P_{s1}^T \ge |h_2|^2 P_r^T$$
 and $|h_2|^2 P_{s2}^T \le |h_1|^2 P_r^T$:

With this assumption, P_r^T and P_{s2}^T determine the capacity expression of the first and second directions, respectively. So the optimum solution is derived as:

$$\frac{B_1}{T_{TWUB}} \le \frac{W}{2} \log_2 \left(1 + \frac{|h_2|^2 P_r^T}{N_0} \right) \to P_{r\min}^T \ge \frac{\left(2^{\frac{2B_1}{WT_{TWUB}}} - 1 \right) N_0}{|h_2|^2},$$
(34a)

$$\frac{B_2}{T_{TWUB}} \le \frac{W}{2} \log_2 \left(1 + \frac{|h_2|^2 P_{s2}^T}{N_0} \right) \to P_{s2\min}^T \ge \frac{\left(2^{\frac{2B_2}{WT_{TWUB}}} - 1 \right) N_0}{|h_2|^2},$$
(34b)

$$P_{s1}^{T} \ge \frac{P_{r}^{T} |h_{2}|^{2}}{|h_{1}|^{2}} \to P_{s1\min}^{T} = \frac{\left(2^{\frac{2\theta_{1}}{WT_{TWUB}}} - 1\right) N_{0}}{|h_{1}|^{2}}.$$
 (34c)

According to the assumption of this state:

$$|h_{2}|^{2}P_{s2}^{T} \leq |h_{1}|^{2}P_{r}^{T} \rightarrow \frac{\left(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{1}|^{2}} \leq \frac{\left(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{2}|^{2}}.$$
(35)

So, again the optimum solution is (31).

4. *if*
$$|h_1|^2 P_{s1}^T \le |h_2|^2 P_r^T$$
 and $|h_2|^2 P_{s2}^T \le |h_1|^2 P_r^T$:

With this assumption, the optimum power of each node can be obtained as follows:

$$\frac{B_1}{T_{TWUB}} \le \frac{W}{2} \log_2 \left(1 + \frac{|h_1|^2 P_{s_1}^T}{N_0} \right) \to P_{s_1 \min}^T \ge \frac{\left(2^{\frac{2B_1}{WT_{TWUB}}} - 1 \right) N_0}{|h_1|^2},$$
(36a)

$$\frac{B_2}{T_{TWUB}} \le \frac{W}{2} \log_2 \left(1 + \frac{|h_2|^2 P_{s2}^T}{N_0} \right)
\rightarrow P_{s2\min}^T \ge \frac{\left(2^{\frac{2B_2}{WT_{TWUB}}} - 1 \right) N_0}{|h_2|^2},$$
(36b)

$$P_{r}^{T} \ge \frac{P_{s1}^{T} |h_{1}|^{2}}{|h_{2}|^{2}} \to P_{r}^{T} \ge \frac{\left(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{2}|^{2}},$$
(36c)

$$P_{r}^{T} \ge \frac{P_{s2}^{T} |h_{2}|^{2}}{|h_{1}|^{2}} \to P_{r}^{T} \ge \frac{\left(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1\right)N_{0}}{|h_{1}|^{2}}.$$
(36d)

Clearly, the optimum solution of the problem (27) is (31) for all possible states. So the CPEE optimization for TWUB is derived as:

$$\begin{array}{ll} \min & T_{TWUB} & T_{TWUB} \left(\frac{P_{\min TWUB}^{T}(T_{TWUB})}{2\varepsilon} + P_{T}^{c} - P_{T}^{ci} \right) + TP_{T}^{ci} \\ s.t: & T_{\min TWUB} \leq T_{TWUB} \leq T. \end{array}$$
(37)

where $T_{\min TWUB}$ is the minimum transmission time. The minimum transmission time for each direction can be derived similar to the OWDF strategy. However, $T_{\min TWUB}$ is the maximum of the minimum transmission times of the both directions.

Problem (37) is a convex optimization problem and can be solved with the methods of [29]. However, if we consider $B_1 = B_2 = B$, analogous to OWDF, the optimum CPEE in low SEs will be derives as:

$$\eta_{EElow}^{TWUB} = \frac{2B}{\frac{BN_0(\ln 2)}{\varepsilon \left|h_{effTWUB}\right|^2 W} 2^{\frac{2B}{WT_{optTWUB}}} + TP_T^{ci}},$$
(38)

where $T_{optTWUB}$ is the optimum transmission time in TWUB and $|h_{effTWUB}|^2$ is defined as:

$$\left|h_{effTWUB}\right|^{2} \triangleq \frac{1}{\frac{1}{|h_{1}|^{2}} + \frac{1}{|h_{2}|^{2}} + \max\left\{\frac{1}{|h_{1}|^{2}}, \frac{1}{|h_{2}|^{2}}\right\}}.$$
(39)

Also, the optimum CPEE in high SEs, where the optimum transmission time is larger than the block duration of T, is obtained as:

$$\eta_{EEhigh}^{TWUB} = \frac{2B}{T\left(\frac{\left(2^{\frac{2B}{2}}-1\right)N_0}{2\varepsilon\left|h_{effTWUB}\right|^2} + P_T^c\right)}.$$
(40)

4.2 CPEE optimization of TWFDF

Functional decode and forward scheme is based on lattice codes and it was first proposed in [18]. In TWFDF, s_1 and s_2 encode their data using nested lattice code and transmit

them to the relay. After receiving the superimposed signal of s_1 and s_2 , node *R* decodes the modulo-sum of the information bits, instead of decoding the entire data, then encodes the sum value and broadcasts it in the second phase [19]. The capacity expression in each direction is defined as [19]:

$$C_{1TWFDF} = \frac{B_1}{T_{TWFDF}} = \frac{W}{2} \min\left\{ \log_2\left(\frac{1}{2} + \frac{|h_1|^2 P_{s1}^T}{N_0}\right), \\ \log_2\left(1 + \frac{|h_2|^2 P_r^T}{N_0}\right)\right\},$$
(41)

$$C_{2TWFDF} = \frac{B_2}{T_{TWFDF}} = \frac{W}{2} \min\left\{ \log_2\left(\frac{1}{2} + \frac{|h_2|^2 P_{s2}^T}{N_0}\right), \\ \log_2\left(1 + \frac{|h_1|^2 P_r^T}{N_0}\right)\right\}.$$
(42)

The second step of the CPEE optimization in TWFDF is:

$$\begin{array}{ll}
\min_{\substack{P_{s_1}^T, P_{s_2}^T, P_r^T \\ s.t}} & P_{s_1}^T + P_{s_1}^T + P_{s_2}^T + P_r^T \\
& P_{s_1}^T \le P_{\max}^t, P_{s_2}^T \le P_{\max}^t, P_r^T \le P_{\max}^t, (41), (42).
\end{array}$$
(43)

Following the analogous procedure, four possible states are considered to solve (43). We just explain the first state. Other possible states can be solved, similarly.

1. if
$$0.5 + |h_1|^2 P_{s1}^T / N_0 \ge 1 + |h_2|^2 P_r^T / N_0$$
 and $0.5 + |h_2|^2 P_{s2}^T / N_0 \ge 1 + |h_1|^2 P_r^T / N_0$:

To achieve desired bit rates of C_{1TWFDF} and C_{2TWFDF} the minimum transmission power of each node is obtained as:

$$\frac{B_1}{T_{TWFDF}} \le \frac{W}{2} \log_2 \left(1 + \frac{|h_2|^2 P_r^T}{N_0} \right)
\rightarrow P_r^T \ge \frac{\left(2^{\frac{2B_1}{WT_{TWFDF}}} - 1 \right) N_0}{|h_2|^2},$$
(44a)

$$\frac{B_2}{T_{TWFDF}} \leq \frac{W}{2} \log_2 \left(1 + \frac{|h_1|^2 P_r^T}{N_0} \right)
\rightarrow P_r^T \geq \frac{\left(2^{\frac{2B_2}{WT_{TWFDF}}} - 1 \right) N_0}{|h_1|^2},$$
(44b)

$$P_{r\min}^{T} = \max\left\{\frac{\left(2^{\frac{2B_{1}}{WT_{TWFDF}}} - 1\right)N_{0}}{\left|h_{2}\right|^{2}}, \frac{\left(2^{\frac{2B_{2}}{WT_{TWFDF}}} - 1\right)N_{0}}{\left|h_{1}\right|^{2}}\right\},\tag{44c}$$

$$P_{s1}^{T} \ge \frac{N_{0}}{2|h_{1}|^{2}} + \frac{P_{r}^{T}|h_{2}|^{2}}{|h_{1}|^{2}} \to P_{s1\min}^{T} = \frac{N_{0}}{2|h_{1}|^{2}} + \frac{P_{r}^{T}|h_{2}|^{2}}{|h_{1}|^{2}},$$

$$(44d)$$

$$P_{s2}^{T} \ge \frac{N_{0}}{2|h_{2}|^{2}} + \frac{P_{r}^{T}|h_{1}|^{2}}{|h_{2}|^{2}} \to P_{s2\min}^{T} = \frac{N_{0}}{2|h_{2}|^{2}} + \frac{P_{r}^{T}|h_{1}|^{2}}{|h_{2}|^{2}}.$$

$$(44e)$$

With the help of "Appendix", the optimum solution for (43) is obtained as:

$$P_{\text{minTWFDF}}^{T}(T_{TWFDF}) = P_{s1\min}^{T} + P_{s2\min}^{T} + P_{r\min}^{T},$$

$$P_{\text{minTWFDF}}^{T}(T_{TWFDF})$$

$$= \frac{N_{0}}{2|h_{effOWDF}|^{2}} + \frac{\left(2^{\frac{2B_{1}}{WT_{TWFDF}}} - 1\right)N_{0}}{|h_{1}|^{2}} + \frac{\left(2^{\frac{2B_{2}}{WT_{TWFDF}}} - 1\right)N_{0}}{|h_{2}|^{2}} + \max\left\{\frac{\left(2^{\frac{2B_{1}}{WT_{TWFDF}}} - 1\right)N_{0}}{|h_{2}|^{2}}, \frac{\left(2^{\frac{2B_{2}}{WT_{TWFDF}}} - 1\right)N_{0}}{|h_{1}|^{2}}\right\}.$$
(45)

For other possible states also the optimum solution is (45). So the CPEE optimization for TWFDF is derived as:

$$\begin{array}{ll} \min & T_{TWFDF} & T_{TWFDF} \left(\frac{P_{\min TWFDF}^{T}(T_{TWFDF})}{2\varepsilon} + P_{T}^{c} - P_{T}^{ci} \right) + TP_{T}^{ci} \\ s.t: & T_{\min TWFDF} \leq T_{TWFDF} \leq T. \end{array}$$
(46)

where $T_{\min TWFDF}$ is the minimum transmission time and it is obtained by substituting the maximum transmission power in (41) and (42) similar to OWDF and TWUB.

Problem (46) is a convex optimization problem. If we consider $B_1 = B_2 = B$, similar to the pervious strategies, the optimum CPEE in low SEs is derived as:

$$\eta_{EElow}^{TWFDF} = \frac{2B}{\frac{BN_0(\ln 2)}{\varepsilon \left|h_{effTWUB}\right|^2 W} 2^{\frac{2B}{WT_{optTWFDF}}} + TP_T^{ci}},$$
(47)

where $T_{optTWFDF}$ is the optimum transmission time in TWFDF. Also the optimum CPEE in high SEs can be obtained as:

$$\eta_{EEhigh}^{TWFDF} = \frac{2B}{T\left(\frac{N_0}{4\left|h_{eff}OWDF\right|^2} + \frac{\left(2^{\frac{2R}{TW}} - 1\right)N_0}{2\varepsilon\left|h_{eff}TWUB\right|^2} + P_T^c\right)}.$$
(48)

4.3 CPEE optimization of TWDF

In TWDF, the relay node completely decodes the messages sent from s_1 and s_2 , then uses XOR scheme or superposition coding (SPC) to form the new message and broadcasts this coded information bits in the second phase. The capacity expression in each direction for TWDF is derived as [19]:

$$C_{1TWDF} = \frac{B_1}{T_{TWDF}} = \frac{W}{2} \min\left\{\frac{1}{2}\log_2\left(1 + \frac{2P_{s1}^t|h_1|^2}{N_0}\right), \\ \log_2\left(1 + \frac{|h_2|^2 P_r^t}{N_0}\right)\right\},$$
(49)

$$C_{2TWDF} = \frac{B_2}{T_{TWDF}} = \frac{W}{2} \min\left\{\frac{1}{2}\log_2\left(1 + \frac{2P_{s2}^t |h_2|^2}{N_0}\right), \\ \log_2\left(1 + \frac{|h_1|^2 P_r^t}{N_0}\right)\right\}.$$
(50)

Analogues to TWUB and TWFDF, four possible states are considered. The minimum summation of transmission powers for TWDF is obtained as:

$$P_{\min TWDF}^{I}(T_{TWDF}) = P_{s1\min}^{I} + P_{s2\min}^{I} + P_{r\min}^{I},$$

$$P_{\min TWDF}^{I}(T_{TWDF})$$

$$= \frac{\left(2^{\frac{4B_{1}}{WT_{TWDF}}} - 1\right)N_{0}}{2|h_{1}|^{2}} + \frac{\left(2^{\frac{4B_{2}}{WT_{TWDF}}} - 1\right)N_{0}}{2|h_{2}|^{2}}$$

$$+ \max\left\{\frac{\left(2^{\frac{2B_{1}}{WT_{TWDF}}} - 1\right)N_{0}}{|h_{2}|^{2}}, \frac{\left(2^{\frac{2B_{2}}{WT_{TWDF}}} - 1\right)N_{0}}{|h_{1}|^{2}}\right\}.$$
(51)

If we consider $B_1 = B_2 = B$, similar to the pervious strategies, the optimum CPEE in low SEs is derived as:

$$\eta_{EElow}^{TWDF} = \frac{2B}{\frac{BN_0(\ln 2)}{\varepsilon \left|h_{eff} \circ WDF\right|^2 W}} 4^{\frac{2B}{WT_{opt}TWDF}} + \frac{BN_0(\ln 2)}{\varepsilon \left|h_{eff} TWDF\right|^2 W} 2^{\frac{2B}{WT_{opt}TWDF}} + TP_T^{ci},$$
(52)

where $T_{optTWDF}$ is the optimum transmission time in TWDF and $|h_{effTWDF}|^2$ is defined as:

$$|h_{effTWDF}|^2 \triangleq \frac{1}{\max\left\{\frac{1}{|h_1|^2}, \frac{1}{|h_2|^2}\right\}}.$$
 (53)

Also the optimum CPEE in high SEs can be obtained as:

$$\eta_{EEhigh}^{TWDF} = \frac{2B}{T\left(\frac{\left(2^{\frac{4B}{WT}}-1\right)N_0}{4\varepsilon\left|h_{eff}OWDF\right|^2} + \frac{\left(2^{\frac{2B}{WT}}-1\right)N_0}{2\varepsilon\left|h_{eff}TWDF\right|^2} + P_T^{ci}\right)}.$$
(54)

5 Two-way DF relaying with three phases

In this section, the CPEE optimization of two-way DF relaying with three phases (TW3DF) is developed. This strategy consists of three phases [30]. According to Fig. 1(d), in the first phase, node s_1 sends B_1 information bits to the relay node R. In the second phase, node s_2 sends B_2 information bits to the relay node. Then, in the third phase node R broadcasts its message to both sources. It is assumed that the system spends $\frac{T_{TW3DF}}{3}$ seconds in each phase. Obviously, $T_{TW3DF} \leq T$, so the system is in idle mode for $T - T_{TW3DF}$ seconds. The CPEE and energy consumption of the system in two-way scenario with three phases are derived as:

$$\eta_{EETW3DF} = \frac{B_1 + B_2}{E_{TW3DF}},\tag{55}$$

$$E_{TW3DF} = \frac{T_{TW3DF}}{3} \left(\frac{P_{s1}^{T}}{\epsilon} + P_{s1}^{ct} + P_{r}^{cr} \right) + \frac{T_{TW3DF}}{3} \left(\frac{P_{s2}^{T}}{\epsilon} + P_{s2}^{ct} + P_{r}^{cr} \right) + \frac{T_{TW3DF}}{3} \left(\frac{P_{r}^{T}}{\epsilon} + P_{r}^{ct} + P_{s1}^{cr} + P_{s2}^{cr} \right) + (T - T_{TW3DF})(P_{s1}^{ci} + P_{s2}^{ci} + P_{r}^{ci}), E_{TW3DF} = T_{TW3DF} \left(\frac{P_{s1}^{T} + P_{s2}^{T} + P_{r}^{T}}{3\epsilon} + P_{T3}^{c} - P_{T}^{ci} \right) + TP_{T}^{ci},$$
(56)

where $P_{T3}^c \triangleq \frac{(P_{s1}^{ct} + P_{s2}^{ct} + P_{s1}^{ct} + P_{s2}^{cr} + 2P_{r}^{cr})}{3}$, $P_T^{ci} \triangleq P_{s1}^{ci} + P_{s2}^{ci} + P_{r}^{ci}$. So the optimization problem is obtained as follow:

$$\begin{array}{ll} \min_{\substack{T_{TW3DF}, P_{s1}^T, P_{s2}^T, P_r^T \\ s.t}} & T_{TW3DF} \left(\frac{P_{s1}^T + P_{s2}^T + P_r^T}{3\varepsilon} + P_{T3}^c - P_T^{ci} \right) + TP_T^{ci} \\ & T_{TW3DF} \leq T, P_{s1}^T \leq P_{\max}^t, P_{s2}^T \leq P_{\max}^t, P_r^T \leq P_{\max}^t. \end{array}$$

$$(57)$$

In this strategy, the source nodes do not send their information simultaneously to the relay node, as a result, the system does not have multiplexing loss [31]. The capacity expressions in each direction are derived as:

$$C_{1TW3DF} = \frac{B_1}{T_{TW3DF}} = \frac{W}{3} \min\left\{\log_2\left(1 + \frac{|h_1|^2 P_{s1}^T}{N_0}\right), \\ \log_2\left(1 + \frac{|h_2|^2 P_r^T}{N_0}\right)\right\},$$
(58)

$$C_{2TW3DF} = \frac{B_2}{T_{TW3DF}} = \frac{W}{3} \min\left\{\log_2\left(1 + \frac{|h_2|^2 P_{s_2}^T}{N_0}\right), \\ \log_2\left(1 + \frac{|h_1|^2 P_r^T}{N_0}\right)\right\}.$$
(59)

The minimum of power summation of the nodes can be obtained similar to TWUB strategy as follows:

$$P_{\min TW3DF}^{T}(T_{TW3DF}) = P_{s1\min}^{T} + P_{s2\min}^{T} + P_{r\min}^{T},$$

$$P_{\min TW3DF}^{T}(T_{TW3DF})$$

$$= \frac{\left(2^{\frac{3B_{1}}{WT_{TW3DF}}} - 1\right)N_{0}}{|h_{1}|^{2}} + \frac{\left(2^{\frac{3B_{2}}{WT_{TW3DF}}} - 1\right)N_{0}}{|h_{2}|^{2}} + \max\left\{\frac{\left(2^{\frac{3B_{1}}{WT_{TW3DF}}} - 1\right)N_{0}}{|h_{2}|^{2}}, \frac{\left(2^{\frac{3B_{1}}{WT_{TW3DF}}} - 1\right)N_{0}}{|h_{1}|^{2}}\right\}.$$
(60)

Analogous to other two-way strategies, if we consider $B_1 = B_2 = B$, the optimum CPEE in low SEs is derived as:

$$\eta_{EElow}^{TW3DF} = \frac{2B}{\frac{BN_0(\ln 2)}{\varepsilon \left|h_{effTWJB}\right|^2 W} 2^{\frac{3B}{WT_{optTW3DF}}} + TP_T^{ci}},$$
(61)

where $T_{optTW3DF}$ is the optimum transmission time in TW3DF strategy. Also, the optimum CPEE in high SEs, where the optimum transmission time is larger than the block duration of *T*, is obtained as:

$$\eta_{EEhigh}^{TW3DF} = \frac{2B}{T\left(\frac{\left(2^{\frac{2B}{2TW}}-1\right)N_0}{3c|h_{effTWUB}|^2} + P_{T3}^c\right)}.$$
(62)

6 Simulation and results

In this section, simulations are presented to verify our findings, explore the trade offs, and compare the DF and AF scheme in one- and two-way relaying. It is assumed that three nodes are located on a straight line. Channels are assumed to be Rayleigh block fading channels. The distance between two sources is assumed to be 200(m) and the relay position is in the middle of the two sources. The path loss is modelled as $30 + 10 \log_{10}(dis \tan ce^{\alpha})$ dBm, where α is the attenuation factor. Other parameters are assumed as: bandwidth for each user W = 10 MHz, block duration time T = 5 mS, maximum available transmission power in each node $P_{max}^{t} = 40$ dBm, noise power density $N_0 = -104$ dBm and power amplifier efficiency $\varepsilon = 0.35$. From [25], the circuit power in practical systems ranges from dozens to hundreds of mW, and the circuit power is set in this range for simulations.

In each simulation, the optimal EE for a given SE is obtained according to the previous sections (optimal EE for the AF scheme and the direct transmission are given in [25]) and then the optimal EE versus SE graph is depicted. According to these simulations, it can be found out that how optimal EE varies as a function of SE in different strategies. All the results are averaged over 1000 channel realizations. For all of the strategies, it is assumed that $B_1 = B_2 = B$.

Similar to [25], we consider an AWGN channel, where $E(|h_{s1s2}|^2)$ is normalized to one and the distance from *R* to nodes s_1 and s_2 are respectively, *d* and 1 - d. With the following assumptions: $E(|h_1|^2) = 1/(d)^{\alpha}$ and $E(|h_2|^2) = 1/(1 - d)^{\alpha}$. The equivalent channel gain for each strategy can be approximated as:

$$E(|h_{effAF}|^2) \approx E\left(\frac{1}{\left(\frac{1}{|h_1|} + \frac{1}{|h_2|}\right)^2}\right) \approx \left(\frac{1}{d^{\frac{2}{2}} + (1-d)^{\frac{2}{2}}}\right)^2,$$
(63a)

$$E(|h_{effOWDF}|^2) \approx E\left(\frac{1}{\frac{1}{|h_1|^2} + \frac{1}{|h_2|^2}}\right) \approx \frac{1}{d^{\alpha} + (1-d)^{\alpha}},$$
 (63b)

$$E(|h_{effTWUB}|^{2}) \approx E\left(\frac{1}{\frac{1}{|h_{1}|^{2}} + \frac{1}{|h_{2}|^{2}} + \max\left\{\frac{1}{|h_{1}|^{2}}, \frac{1}{|h_{2}|^{2}}\right\}}\right)$$
$$\approx \frac{1}{d^{\alpha} + (1-d)^{\alpha} + \max\left\{d^{\alpha}, (1-d)^{\alpha}\right\}},$$
(63c)

$$E(|h_{effTWDF}|^{2}) \approx E\left(\frac{1}{\max\{\frac{1}{|h_{1}|^{2}}, \frac{1}{|h_{2}|^{2}}\}}\right)$$
$$\approx \frac{1}{\max\{d^{\alpha}, (1-d)^{\alpha}\}}.$$
(63d)

These approximations for equivalent channels are helpful to analyse and compare EEs for different strategies in high SEs. In low SEs, because of two reasons we cannot use analytical results and therefore simulation results are needed to compare the CPEE for different strategies. First, the closed form solution of optimum transmission times are not obtained and second, in low SEs the circuitry power of nodes are comparable to the transmission powers and it makes the analytical comparisons very difficult. In all figures of the simulation results, the X-axis is the number of overall transmitted bits in both directions normalized by block duration of T and bandwidth W.

In Fig. 2, the optimum CPEE versus SE for TWFDFD, OWDF and DT are depicted for different α . Clearly, decrease in value of α leads to the improvement of the channel condition and the CPEE for each strategy. However, as it is shown in Fig. 2, the variation of α is more effective on the DT strategy in comparison to the other strategies. For $\alpha = 3$ in low SEs, OWDF is more energy efficient than DT, however, for $\alpha = 2.5$ the CPEE of both strategies increase and become almost equal.



Fig. 2 The comparison of EE–SE with circuit power consideration in TWFDF, OWDF and DT with different path loss attenuations. The circuit power for transmission, reception and idle mode are respectively set as, $P^{ct} = P^{cr} = 10 \text{ mW}$ and $P^{ci} = 10 \text{ mW}$

In high SEs, for a constant α , we can compare the CPEE of different strategies using equivalent channels obtained in (63). In high SEs, we have:

$$\frac{\eta_{EEhigh}^{TWFDF}}{\eta_{EEhigh}^{OWDF}} \approx \frac{\frac{(2^{\frac{2W}{W}}-1)N_0T}{2\varepsilon |h_{eff} \otimes WDF}|^2}}{\frac{N_0T}{4\varepsilon |h_{eff} \otimes WDF}|^2 + \frac{(2^{\frac{2B}{WT}}-1)N_0T}{2\varepsilon |h_{eff} T W UB}|^2}} = \frac{|h_{eff} T W UB|^2 (2^{\frac{4B}{WT}}-1)}{|h_{eff} \otimes WDF}|^2 (2^{\frac{2B}{WT}}-1)} = \frac{2(2^{\frac{2B}{WT}}+1)}{3},$$
(64a)

$$\frac{\eta_{EEhigh}^{TWFDF}}{\eta_{EEhigh}^{DT}} \approx \frac{\frac{(2^{\frac{2RT}{4}} - 1)N_0 T}{\epsilon |h_{s1s2}|^2}}{\frac{N_0 T}{4\epsilon |h_{effODF}|^2} + \frac{(2^{\frac{2R}{4}} - 1)N_0 T}{2\epsilon |h_{effTUB}|^2}} = \frac{2 |h_{effTUB}|^2}{|h_{s1s2}|^2} = \frac{2^{\alpha+1}}{3},$$
(64b)

$$\frac{\eta_{EEhigh}^{DT}}{\eta_{EEhigh}^{OWDF}} \approx \frac{|h_{s1s2}|^2 (2^{\frac{4B}{WT}} - 1)}{|h_{effOWDF}|^2 (2^{\frac{2B}{WT}} - 1)} = \frac{(2^{\frac{2B}{WT}} + 1)}{2^{\alpha}}.$$
 (64c)

According to (64a) and (64b), it is clear that in high SEs, TWFDF is more energy efficient than DT and OWDF. Also, (64c) shows that in high SEs for $\alpha = 3$, the CPEE of DT is higher than that of OWDF (in high SEs $\frac{2B}{TW} > 3$ and obviously the expression of (64c) is larger than 1) which is supported by our simulation results.

For $\alpha = 3$, with the help of mathematical analysis and simulation results obtained in Fig. 2, we can conclude that in low SEs, OWDF has the highest and DT has the lowest CPEE performance. However, in high SEs, TWDF, DT and OWDF, respectively have the highest CPEE.



Fig. 3 The comparison of EE–SE with different circuit power considerations in TWUB and OWDF $\$

In Fig. 3, the optimum CPEE of TWUB and OWDF with different circuit power consumptions are depicted. In high SEs, for each strategy, the circuit power consumption is negligible in comparison to the transmission power and does not affect the CPEE curves. Another point is that the OWDF strategy consumes more transmission power comparatively. Therefore, the circuit power consumption has less effect on the CPEE of OWDF in comparison to the CPEE of TWUB and the optimal CPEE curves of OWDF converge towards each other in lower SEs.

In Fig. 4, we compare the CPEEs of different strategies with equal circuit power at each node, where $\alpha = 3$ and the



Fig. 4 The comparison of EE–SE with circuit power consideration in TWUB, TW3DF, TWDF, OWDF, TWFDF, TWAF, OWAF and DT. It is assumed that $\alpha = 3$ and $P^{ct} = P^{cr} = 100 \text{ mW}$ and $P^{ci} = 10 \text{ mW}$

relay node is located in the midpoint. The results for AF strategies agree with the results of [25]. According to [25], TWAF outperforms OWDF and DT for all SEs. In low SEs, OWAF has better performance compared to DT, however, for high SEs the converse is true.

Here, we compare the CPEE of DT, AF and DF strategies. If the equivalent channels, obtained in (63), substitute in the CPEE of different strategies in high SEs, we can compare them to find out when and which relay strategies can be helpful for improving the CPEE. The CPEE of different two-way relaying strategies are compared with each other and DT as follows:

$$\frac{\eta_{EEhigh}^{TWUB}}{\eta_{EEhigh}^{TWFDF}} \approx \frac{\frac{N_0 T}{4\epsilon \left|h_{eff} OWDF\right|^2} + \frac{(2^{\overline{WT}} - 1)N_0 T}{2\epsilon \left|h_{eff} TWUB\right|^2}}{\frac{(2^{\overline{WT}} - 1)N_0 T}{2\epsilon \left|h_{eff} TWUB\right|^2}},$$
(65a)

....

$$\frac{\eta_{EEhigh}^{TWUB}}{\eta_{EEhigh}^{TWAF}} \approx \frac{2\left|h_{effTWUB}\right|^2}{\left|h_{effAF}\right|^2} = \frac{8}{3},$$
(65b)

$$\frac{\eta_{EEhigh}^{TWUB}}{\eta_{EEhigh}^{TW3DF}} \approx \frac{\frac{(2^{\overline{WT}}-1)N_0T}{3\epsilon \left|h_{effTWUB}\right|^2}}{\frac{2B}{(2^{\overline{WT}}-1)N_0T}} = \frac{2(2^{\frac{2B}{WT}}-1)}{3(2^{\frac{2B}{WT}}-1)} > 1$$
(65c)

$$\frac{\eta_{EEhigh}^{TW3DF}}{\eta_{EEhigh}^{TWDF}} \approx \frac{\frac{(2^{\frac{2BT}{W}}-1)N_0T}{4\varepsilon \left|h_{eff}OWDF\right|^2} + \frac{(2^{\frac{2BT}{W}}-1)N_0T}{2\varepsilon \left|h_{eff}TWDF\right|^2}}{\frac{2\varepsilon \left|h_{eff}TWDF\right|^2}{3\varepsilon \left|h_{eff}TWUB\right|^2}} = \frac{3(2^{\frac{2B}{WT}}-1)\left|h_{eff}TWUB\right|^2}{4(2^{\frac{3B}{WT}}-1)\left|h_{eff}OWDF\right|^2} + \frac{3(2^{\frac{2B}{WT}}-1)\left|h_{eff}TWUB\right|^2}{2(2^{\frac{3B}{WT}}-1)\left|h_{eff}TWDF\right|^2} = \frac{(2^{\frac{4B}{WT}}-1)}{2(2^{\frac{3B}{WT}}-1)} + \frac{(2^{\frac{2B}{WT}}-1)}{2(2^{\frac{2B}{WT}}-1)} > 1$$
(65d)

$$\frac{\eta_{EEhigh}^{TW3DF}}{\eta_{EEhigh}^{TWAF}} \approx \frac{\frac{(2\overline{WT}-1)N_0T}{\epsilon \left|h_{effAF}\right|^2}}{\frac{(2\overline{WT}-1)N_0T}{3\epsilon \left|h_{effOWDF}\right|^2}} = \frac{3(2\frac{2B}{WT}-1)\left|h_{effOWDF}\right|^2}{(2\frac{3B}{WT}-1)\left|h_{effAF}\right|^2} = \frac{4(2\frac{2W}{WT}-1)}{(2\frac{3B}{WT}-1)} < 1,$$
(65e)

2B

$$\frac{\eta_{EEhigh}^{TW3DF}}{\eta_{EEhigh}^{DT}} \approx \frac{\frac{(2^{\overline{WT}}-1)N_0T}{\varepsilon}}{\frac{(2^{\overline{WT}}-1)N_0T}{3\varepsilon \left|h_{effTWUB}\right|^2}} = \frac{3(2^{\frac{2B}{WT}}-1)\left|h_{effTWUB}\right|^2}{(2^{\frac{3B}{WT}}-1)} \\ = \frac{2^{\alpha}(2^{\frac{2B}{WT}}-1)}{(2^{\frac{2B}{WT}}-1)} < 1.$$
(65f)

According to (65a) and (65b), the CPEE of TWUB and TWFDF are almost equal for high SEs and they both outperform the CPEE of TWAF. It should be mentioned that in high SEs [more than 6(bits/s/Hz)], we have $\frac{B}{TW} > 3$. Therefore, with respect to (65), it is concluded that in high

SEs, TWUB, TWFDF, TWAF, and DT have better CPEE than TW3DF and TWDF. Also, according to (65d), the CPEE of TW3DF outperforms that of TWDF. Here the CPEE of TWDF and OWDF are compared:

$$\frac{\eta_{EEhigh}^{OWDF}}{\eta_{EEhigh}^{TWDF}} \approx \frac{\frac{(2^{\frac{48}{WT}}-1)N_0T}{4\epsilon|h_{eff}OWDF|^2} + \frac{(2^{\frac{2W}{WT}}-1)N_0T}{2\epsilon|h_{eff}TWDF|^2}}{\frac{(2^{\frac{48}{WT}}-1)N_0T}{2\epsilon|h_{eff}OWDF|^2}} = \frac{1}{2} + \frac{\left|h_{eff}OWDF\right|^2}{(2^{\frac{28}{WT}}+1)\left|h_{eff}TWDF\right|^2}$$
$$= \frac{1}{2} + \frac{1}{2(2^{\frac{28}{WT}}+1)}.$$
(66)

According to (66) and (65d), it is clear that TWDF and TW3DF have better performance than OWDF in high SEs. Also, the CPEE of OWDF and AF relaying strategies in high SEs can be compared as follows:

$$\frac{\eta_{EEhigh}^{OWDF}}{\eta_{EEhigh}^{OWAF}} \approx \frac{\left|h_{effOWDF}\right|^{2}}{\left|h_{effAF}\right|^{2}} = 2,$$
(67a)

$$\frac{\eta_{EEhigh}^{OWDF}}{\eta_{EEhigh}^{TWAF}} \approx \frac{2 \left| h_{effOWDF} \right|^2 (2^{\frac{2B}{WT}} - 1)}{\left| h_{effAF} \right|^2 (2^{\frac{4B}{WT}} - 1)} = \frac{4}{(2^{\frac{2B}{WT}} + 1)}.$$
 (67b)

According to (67a) and (67b), in high SEs, the CPEE performance of OWDF is better than OWAF, however, the CPEE of TWAF outperforms the CPEE of OWDF.

With the help of mathematical analysis for high SEs (65–67) and simulation results obtained in Fig. 4, it can be concluded that TWUB has the best performance for all SEs. In low SEs, the CPEE curve of TW3DF is close to the CPEE curve of TWUB and both of them outperform other strategies. However, in high SEs, TWFDF has similar



Fig. 5 Outage probability versus SE with circuit power consideration in TWUB, TW3DF, TWDF, OWDF, TWFDF, TWAF, OWAF and DT. It is assumed that $\alpha = 3$ and $P^{ct} = P^{cr} = 100 \text{ mW}$ and $P^{ci} = 10 \text{ mW}$

performance to TWUB. The CPEE of OWDF in low SEs is almost high and outperforms all strategies except TWUB, TW3DF and TWDF, however, in high SEs, the CPEE of all two-way strategies (including AF and DF) and DT are better than that of OWDF. It should be mentioned that TW3DF outperforms TWDF for all SEs. The CPEE of TWAF, in low SEs, is lower than that of all DF relaying strategies (including one-way and two-way), however, in high SEs, it has better performance than OWDF, TWDF, and TW3DF.

Figure 5 shows the outage probability versus SE for different strategies. In each strategy, when the information bits cannot be sent, even if all the nodes use their maximum power in block duration of *T*, outage occurs. As it is shown in Fig. 4, when $\alpha = 3$, the outage probability of TWAF, TWFDF, TWUB and DT is close to zero for all SEs. However, for high SEs, more than 5 or 5.5 (bits/s/Hz), the outage probability of OWAF, OWDF and TWDF exceeds the acceptable threshold, say 10 % according to [25]. Also, the outage probability ofTW3DF is less than 0.1 for all SEs.

7 Conclusion

In this paper, the energy efficiency optimization problem in a cooperative relay network with sleep mode is considered. The optimum energy efficiency of OWDF, TWUB, TWFDF, TWDF, and TW3DF strategies are derived by optimizing the transmission power and time.

Analytical and simulation results showed that in symmetric systems predictably TWUB has the best energy efficiency for all SEs. In low SEs, TW3DF, TWDF, OWDF, TWFDF, TWAF, OWAF and DT have the highest energy efficiencies, respectively. However, in high SEs, TWFDF has similar performance to the TWUB and TWAF is the third energy efficient strategy and DT outperforms TW3DF, TWDF, OWDF and OWAF strategies. It can be concluded that in low SEs, the CPEE of DF relying strategies (including of one-way and two-way) outperform AF strategies and DT. On the other hand, in high SEs, TWAF and DT are more energy efficient than TWDF, TW3DF, and OWDF. Consequently, the comparison results reveal that the use of DF or AF relaying strategies is not always more energy efficient than DT. The system should consider the SE condition and channel statics to choose the most energy efficient strategy.

Also, the impact of different channel conditions and circuitry power on the energy efficiency of DF relaying strategies is shown in our simulation results. Furthermore, the outage probability of different strategies is compared with each other in different SEs.

Appendix

In this Appendix, it is explained how (30) can be written as (31) in details. We consider two possible states to clear the proof:

1. If
$$(2^{\frac{2B_1}{WT_{TWUB}}} - 1)/|h_2|^2 \ge (2^{\frac{2B_2}{WT_{TWUB}}} - 1)/|h_1|^2$$
:

With this assumption (30) is written as:

$$P_{s1\min}^{T} + P_{r\min}^{T} + P_{s2\min}^{T} \ge \frac{(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1)N_{0}}{|h_{1}|^{2}} + \frac{(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1)N_{0}}{|h_{2}|^{2}} + \frac{|h_{1}|^{2}(2^{\frac{2B_{1}}{WT_{TWUB}}} - 1)N_{0}}{|h_{2}|^{4}}.$$
 (68)

If we substitute the assumption $|h_1|^2/|h_2|^2 \ge (2^{\frac{2B_2}{WT_{TWUB}}} - 1)/(2^{\frac{2B_1}{WT_{TWUB}}} - 1)$ in (68), the minimum summation of the

transmission powers is derived as (31).
2. if
$$(2^{\frac{2B_1}{WT_{TWUB}}} - 1)/|h_2|^2 \le (2^{\frac{2B_2}{WT_{TWUB}}} - 1)/|h_1|^2$$
:

With this assumption (30) is written as:

$$P_{s1\min}^{t} + P_{r\min}^{t} + P_{s2\min}^{t} \ge \frac{|h_{2}|^{2} (2^{\frac{2B_{2}}{WT_{TWUB}}} - 1)N_{0}}{|h_{1}|^{4}} + \frac{(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1)N_{0}}{|h_{1}|^{2}} + \frac{(2^{\frac{2B_{2}}{WT_{TWUB}}} - 1)N_{0}}{|h_{2}|^{2}}.$$
(69)

If we substitute the assumption $|h_2|^2/|h_1|^2 \ge (2^{\frac{2B_1}{WT_{TWUB}}} - 1)/(2^{\frac{2B_2}{WT_{TWUB}}} - 1)$ in (69), the minimum summation of the transmission powers is obtained as (31).

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